

SOLITON AND PHYSICS

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Soliton - a solitary structurally stable wave with properties of a particle, was discovered in the first half of the XIX century and initially did not attract any special attention. In those far years hardly someone could assume that over time it will become more and more often object of serious researches and will take particular and very important place in a science about the Nature. Now it becomes quite obvious that the theory of soliton is not only a basis for many "macrocosm" processes study but also for substance research at the level of elementary particles.

The problem is not simply that solitons possess very unusual properties of a wave and a particle at the same time (hold the shape at propagation and even after collision with each other). It appeared also that the description of the physical processes underlying this very beautiful and unusual phenomenon, demands use of the specific nonlinear equations. Therefore solitons gave rise to development of the whole areas of the applied mathematics devoted to numerous and various manifestations of these nonlinear processes.

Wide application of mathematical methods into physics is natural and very useful. However it is impossible to overlook that the clear physical treatment is also necessary and not less useful. In a case with solitons the mathematics obviously prevails over physics. Moreover, many students risk being disappointed in the first attempt to get acquainted with solitons, not finding the plain and quite clear description of physics of this phenomenon.

Greatest "revelation" at the explanation of the physical mechanism, for example, of the very first opened by Russell in 1834 soliton (on a water surface) is the instruction on balance of two contradictory factors [1]: *lonely waves are formed, when the effects of nonlinearity, doing a wave more steeply, are counterbalanced by effects of the dispersion, tending to make it flat (i.e. "wash away" it).*

As a whole this is correct explanation, but soliton itself "somehow" creates these effects («the point grinned and became a comma ...»).

At first we will quote the soliton («wave of translation») description, made by its opener John Scott Russell in «The report on waves», which has become classical [2]: « I was observing the motion of a boat which was rapidly drawn along a narrow channel by a pair of horses, when the boat suddenly stopped - not so the mass of water in the channel which it had put in motion; it accumulated round the prow of the vessel in a state of violent agitation, then suddenly leaving it behind, rolled forward with great velocity, assuming the form of a large solitary elevation, a rounded, smooth and well-defined heap of water, which continued its course along the channel apparently without change of form or diminution of speed. I followed it on horseback, and overtook it still rolling on at a rate of some eight or nine miles an hour, preserving its original figure some thirty feet long and a foot to a foot and a half in height. Its height gradually diminished, and after a chase of one or two miles I lost it in the windings of the channel. Such, in the month of August 1834, was my first

chance interview with that singular and beautiful phenomenon which I have called the Wave of Translation».

In addition to well known complexity of the mathematical description there is also other reason for extensiveness of the theory of this phenomenon: solitons appear really many-sided. Therefore also physical mechanisms of solitons of different types appear very unlike, though are characterized by some common features of the mathematical description. To such very different solitons it is possible to attribute, for example, impulses in nervous fibers and powerful light impulses in the nonlinear medium.

Moreover, some kinds of solitons raise doubts in "purity" of its belonging to this class of the physical phenomena, while other their "representatives" can smoothly convert themselves from "simple" waves to solitons.

Analyzing electromagnetic rotating soliton - "building block" of internal structure of elementary particles [3], I became sure that this soliton version also very differs from other "colleagues": and on its properties, and by description possibilities, and on the physical mechanism.

Here it is necessary to emphasize, nevertheless, one very important property, which is general for all the solitons. Coming to this or that place, soliton changes milieu (saturates it with energy), but leaving this place, it returns everything in an initial condition. However, it is not all the "truth". The fact is that *leaving unchanging the medium, soliton, nevertheless, makes important changes in surrounding space*. Sometimes it is revealed in phase ratios, but more often there are notable "material" changes. On water - it "drags" with itself the corresponding volume of water (moving in the form of a dislocation, or in the form of a loop on a string, also leads to substance shift). If it is electromagnetic soliton in the form of an elementary particle, it, moving, changes charge distribution in space. Thus, as well as any other soliton, it initially fills vacuum with some kind of energy (changes its properties, creating nonzero field divergence), and, when leaving this place, takes away energy and «puts everything in order». We will yet speak at greater length about these soliton physical properties.

Despite all solitons variety, the first soliton, opened by Russell, remains as though a standard of this physical phenomenon. And speaking about solitons, we usually first of all keep in mind the above cited description of the event which has occurred in 1834.

Therefore, considering features of the physical mechanism of the phenomenon, it is logical to return to this historically first kind of solitons. Let's try to look using minimum of mathematics at this physical phenomenon from different points of view. Let's try «to get acquainted closer» with solitons, involving the most unusual analogies and without being afraid of radical simplifications to achieve better understanding.

Soliton on a rope

In literature about solitons usually do not mention a single wave which can be "started up" along the most usual rope stretched on the ground. And such "caution" can be understood, as this example is evident, but demands attentive research and, generally speaking, is not quite correct.

Nevertheless, we will use this example, we will analyze it, and at the same time we will clarify, in what circumstances the mentioned incorrectness appears.



Fig. 1:
The single wave running along a rope stretched on the ground.

So, «everyone who wants» can receive a single wave in the simplest way. It arises and moves along the most customary rope lying on the ground if its end with a fast movement to lift and at once to lower. The running single wave (fig. 1) holds long enough the shape (it seems that if there would be no energy losses and not finite length of a rope, running "hump" could exist for indefinite time).

The wave in the form of such "hillock", very reminding structurally steady lonely wave - soliton, despite seeming simplicity, is mathematically quite difficult described. But we do not just care about accuracy. On the contrary, having even more simplified a task, we will be convinced that the physical picture of the phenomenon becomes just clearer and more evident. However, as it is surprising, even at such simplifications quantitative assessment appears quite acceptable.

Let's begin with the most general principle of soliton existence.

As well as for any wave process, it consists in interaction and mutual transformation of different kinds of energy. In this case it is about interaction of kinetic energy with potential (gravitational) energy of a wave. The process at all its complexity submits to a simple stipulation, reasonable for any wave process. It is that *the general potential energy and the general kinetic energy of the entire wave should equal each other.*

For the approximate analysis we will simplify a form of a single wave (fig. 2). If to imagine soliton movement proceeding together with the circles approximating its form (as it is shown in drawing), to analyze its properties will not present any difficulty.

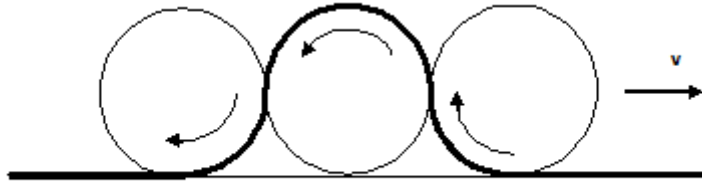


Fig. 2:
Soliton form approximation by circles.

Really, the top half of a wave is formed by the piece of a rope "sliding" on the top half of the central circle with a speed of soliton movement, and two lower parts of a wave (everyone on a circle quarter) "slide" with the same speed along left and right circles. If mentally to combine the top half of the central circle and two quarters of the left and right circles, it will turn out that within a wave it is possible to model *a rope tension* by a rotating ring (fig. 3).



Fig. 3:
Rope tension within a single wave
it is possible to model by a rotating ring.

However, the left and right lower parts of a wave would need to be changed places, and the direction of movement of the top part to be changed on the opposite direction. But *the rope tension in a place of passing of a wave really reminds tension in a rolling ring*. What is the essence of this effect?

Let's look at the process out of the coordinates system moving together with a wave. At once becomes clear that the lifting force supporting a wave is the centrifugal force operating in the top part of a wave. Really, in this part movement of a rope occurs on an arch which camber is directed up, causing the existence of "elevating" force.

At the same time the lower part of a rotating ring (two quarters of a circle) pulls down, counterbalancing centrifugal forces of the top part of a wave. Actually all this very much reminds the inner tube of a bicycle wheel. Only instead of the compressed air the centrifugal forces operating extensively in this case work. Naturally, when the ring quicker moves, the quicker it rotates, and the action of the centrifugal forces is strengthening, so the effect is stronger - reminding "a beefy wheel» (fig. 4).

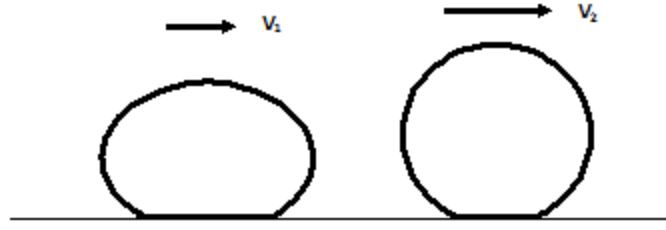


Fig. 4:
**The more the speed of a ring is (v_2 is more than v_1),
so more "pumped up" it seems.**

Speaking about a possible incorrectness of identification of the single wave on a rope as soliton, I just meant a possibility of violation of the main soliton rule. For example, friction force fixing the rope on the ground will hinder the "automatic" increase in height of a wave if kinetic energy of a wave will exceed potential energy. That is, there can be a situation similar to the described above with quickly rolling pliable ring. The more there will be a speed of movement, the rope tension will be stronger, but the corresponding increase in height of a wave will not occur.

At the same time, even the rolling rope ring at a small speed (when it strongly gives) reminds (fig. 4) soliton - under condition of equality to each other of internal kinetic and internal potential energy of this ring that is provided with the "correct" ratio of height of this peculiar wave with its speed of movement. Therefore, *returning to the description of a single wave on a rope, we do not stand for real experiences with inevitable "flaws", but for mental ideal experiments.*

Let's count now kinetic and potential energy of the wave running on a rope, taking into consideration the simplest approximation of its form. In fig. 5 are shown four parts of a rope (within a wave) which have equal kinetic energy. Therefore, if speed of a wave to denote by v , mass of one running meter of a rope - m and circle radius - R , then kinetic energy will be:

$$W_k = 4W_1 = 4 \int_0^{\pi/2} \frac{Rmv^2}{2} ((\cos\alpha)^2 + (1 - \sin\alpha)^2) d\alpha = mRv^2(2\pi - 4). \quad (1)$$

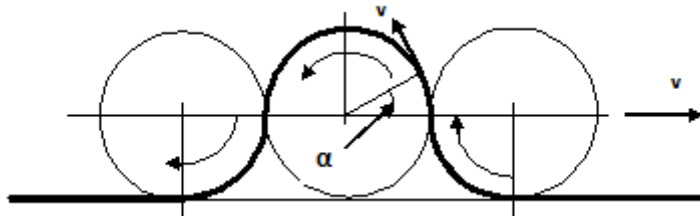


Fig. 5:
**Wave parts with identical kinetic energy
are limited with vertical and horizontal lines.**

At integration the squared speed of each element of length of a rope $Rd\alpha$ was taken as a sum of two squared components (vertical and horizontal velocity).

It is interesting to note that the kinetic energy of a wave appeared to be equal to the doubled kinetic energy of the segment of a rope really transferred by a wave ($2\pi R - 4R$). It is result of multidirectional movement of all the rope length of wave ($2\pi R$).

Potential energy of a piece of the rope forming the wave is defined even more simply: the mass of a ring is multiplied by free fall acceleration and a half of wave height ($H/2 = R$)

$$W_p = (2\pi Rm)gR = 2\pi R^2mg. \quad (2)$$

Taking into account (1) and (2) potential energy will be equal to kinetic energy when speed of movement of a wave is

$$v = \sqrt{\frac{2\pi}{2\pi-4}} \cdot \sqrt{Rg} = \sqrt{\frac{\pi}{2\pi-4}} Hg = 1,173\sqrt{Hg}. \quad (3)$$

Further we will see that this result is close to real value.

Let's discuss the physical mechanism and search for more exact description of the elementary single wave on a rope.

We will pass again to the system of coordinates moving together with a wave. As we already noted, transition from one system of coordinates to another gives the chance to see processes from other point of view and to notice new features.

In this system of coordinates the wave appears motionless, and the rope moves relative to us. Such movement can be modeled by very simple device: the rope is held by two rollers in front and behind of the "wave", and the speed of rope movement is specially regulated (fig. 6).

What will occur at increase of rope speed via such mechanism? The centrifugal forces affecting this «motionless wave» in the form of a loop will increase, the tension of a rope will also increase, and the wave will become higher.



Fig. 6:
At speed increase the centrifugal forces overcome gravity,
and a loop (wave) height raises.

However after achieving some height the increased weight of a loop will counterbalance centrifugal force and balance will be restored at a new level.

In this connection let us remember one more interesting illustration.

It is about a very unusual behavior of a flexible hose through which water is flowing under high pressure. Such hose starts to coil literally «as alive» and escapes from hands. The analogy with a rope pulled through rollers here is obvious, as in the bent hose the centrifugal forces are affecting water moving inside. Moreover, this analogy appears very useful and convenient for the analysis.

The flexible tubing with water moving in it is unstable dynamic system. The fact is that such hose will not motionlessly lie (on a slippery surface) even if to try to stretch it exactly on the ground. The smallest deviation from a straight line will cause at once very strong dynamic force which will entail a hose to move, continuously coiling. It can be proved in a very simple way.

Really, let us suppose that the hose lies motionlessly, but somewhere has a small curvature (fig. 7). If the hose is motionless, we can determine equality of forces acting on a considered small site of a hose: centrifugal force F_c and force of hose tension T . Other forces operating on this considered small site of a hose we will not account. In figure the small slightly bent piece of a hose $R\alpha$ is shown, where R – is major radius of curvature of a hose in this place, and α - very small angle corresponding to small length of the bent site.

The centrifugal force acting on a site of a hose (m - mass of water of one running meter of a hose, mass of the hose we will neglect), is defined as

$$F_c \approx \frac{(m\alpha R)v^2}{R} = mv^2\alpha. \quad (4)$$

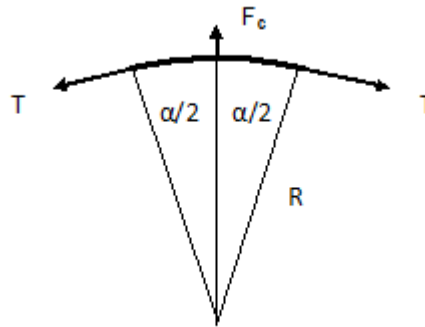


Fig. 7:
Forces acting on a small bend piece of hose.

Centrifugal force is counterbalanced by effective force of a tension

$$F_r = 2T \sin(\alpha/2) \approx T\alpha. \quad (5)$$

Equating (4) and (5), we see that force of a hose tension practically does not depend on a bend angle! That is, it arises with a "jump", and at the slightest curvature the hose will "snatch" out of hands with great forces T depending only on the speed of water supply and its weight on running meter of hose

$$T = mv^2. \quad (6)$$

For this reason hose at high water supply never lies in a quiet state, and it happens even difficult to hold it. Such dynamic instability of a hose with flowing water seriously complicates work of the firefighters compelled constantly to remember this factor, and to make considerable efforts to fix hose nozzle.

The experiment with a rope described above (fig. 6) can be entirely repeated using a hose fixed with rollers, having left free only some part of length of a hose (having the possibility to be bent only vertically up). Then at increase of water supply in a loose part of a hose spontaneously the hose curvature in the form of a loop («a motionless wave») will arise, and its height, eventually, will be a result of an equilibration of centrifugal forces by the weight of the loop.

"Tendency" of the current water to instability is actually observed everywhere. Look more attentively at spring streams. The quiet water current is almost everywhere covered with "motionless waves». These "humps" on the water current as though indicate the roughness of a stream bottom or coasts. But even in trenches with smooth walls and smooth bottom water flow has some instability, and always abounds quite high and almost motionless water "hillocks". This subject we will still discuss.

In the previous reasoning we intentionally idealized occurring processes. Hose tension and water dynamics, obviously, are essentially influenced both by viscosity of water, and its friction on hose walls, as well as by possible turbulence of water movement. Hoses, naturally, also do not possess ideal flexibility. But ideal mental experiments give us the chance to reveal defining features of the physical phenomena.

It is necessary to pay attention to one more essentially important circumstance. Total "elevating" centrifugal force present within a solitary wave (on a rope or on a hose with flowing water) does not depend on a form of the wave and is determined only by a steepness of forward and back fronts of a wave.

This affirmation is based on the fact that a resultant force is equal to the general change of the impulse (of a rope or waters) which occurs, beginning from "entrance" and finishing with "exit" from a wave. That is, we can consider a wave as «a black box» to which for a unit of time a certain impulse goes and then leaves it, having other direction. This resulting change of impulse per unit time is the force counterbalancing «total weight of a wave». As gravity acts vertically down, we should take change of only a vertical component of impulse pro second

$$P = Lmg = 2mv \cdot v \cdot \sin \alpha = 2mv^2 \sin \alpha. \quad (7)$$

Here α - angle of rise (recession) of the forward (back) front of a wave, and L - length of a rope forming a wave.

Thus, the soliton physical mechanism is actually based on the balance of real forces defining process dynamics in the milieu.

Soliton "runs" down a hill!

As we already know, soliton is a wave with properties of a particle. Actually it represents constantly self-reproduced process of interaction of different kinds of energy. Thus soliton «is able to regulate» internal distribution of energy between its different kinds, showing surprising ability to self-organizing in the most unusual environments.

For example, soliton can "run" from a hill (fig. 8). Really, having stretched a rope down on an inclined surface and having started up on it a single wave, it is possible to be convinced that the wave in this case can not only propagate without attenuation, but can even increase its amplitude, increasing at the same time speed of movement. It is obvious that it occurs because of soliton bearing with itself a real part of the rope as we mentioned it earlier. Therefore it spends potential energy of the rope spread out on a hill ("using" its slipping from a hill) for compensation friction energy losses and to *increase its own energy*.



Fig. 8:
Going down a hill, soliton increases internal energy.

Thus soliton, running from a hill, "automatically" increases both kinetic, and the *internal* potential energy.

Let's look more attentively at these processes.

What force affects a wave going down a slope? This force is defined by energy change caused by moving wave

$$F_x = -\frac{dW}{dx}. \quad (8)$$

Therefore, it is defined by *X-projection* of weight of a piece of the rope moving together with a wave

$$F_x = P_x = P \cdot \sin \alpha = (2\pi R - 4R)mg \cdot \sin \alpha. \quad (9)$$

This force produces general increment of wave energy while moving at very small distance dx

$$dW = P_x dx = (2\pi R - 4R)mg \sin \alpha dx. \quad (10)$$

As kinetic energy of a wave is equal to its potential energy, so, proceeding from (2), the general soliton energy is

$$W = 2W_p = 4\pi R^2 mg. \quad (11)$$

Respectively we can define how change of energy of a wave and change of its size (R) are interconnected:

$$dW = 8\pi mg R dR. \quad (12)$$

Equating (10) and (12), we obtain

$$\frac{dR}{dx} = \frac{\pi - 2}{4\pi} \sin \alpha = \text{const}. \quad (13)$$

Thus, the amplitude of a wave will linearly grow with coordinate change.

After getting time derivative (3) and, using (13), we have

$$\frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} = \frac{1}{8} g \sin \alpha = \text{const}. \quad (14)$$

We see that soliton, "rolling down" from a hill, will have *uniform acceleration*, that is, its movement really reminds particle movement. However thus there is an illusion that soliton experiences the free fall acceleration *reduced in eight times*! Such effect is caused by the fact that directed transfer of the rope is rather small in comparison with all the movement of the milieu "involved" in wave process, and also because of soliton distributing rather small energy (received while descending) among two interior kinds of energy.

The similar effect is observed, for example, while wheel (in the form of a ring) descending from a hill. But in this case acceleration decreases only twice because potential energy of a wheel passes to kinetic energy of progress movement and to the "internal" energy - rotation (fig. 9).

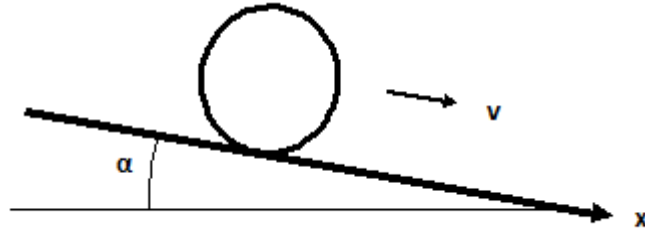


Fig. 9:
Wheel going down the hill can roll or slide.

If the wheel slides on a hill surface (instead of to roll down), its acceleration will be maximum.

Really, change of potential energy of a wheel when moving (displacement dx) both in case of a roll, and in case of sliding is the same

$$dW_p = -P_x dx = -2\pi R mg \sin \alpha dx. \quad (15)$$

On the other hand, kinetic energy and its increment in case when wheel rolls (i.e. taking into account rotation), is as follows

$$W_k = 2\pi Rmv^2, \quad dW_k = 4\pi Rmvdv. \quad (16)$$

Reduction of potential energy is equal to the increment of kinetic energy of a wheel. Therefore, having equated (15) and (16) and after dividing the left and right parts of the received equation into dt (period when these changes occur), we have

$$\frac{dv}{dt} = \frac{1}{2} g \sin \alpha. \quad (17)$$

In case of sliding of a wheel instead of (16) we receive

$$W_k = \frac{2\pi Rmv^2}{2}, \quad dW_k = 2\pi Rmvdv. \quad (18)$$

Respectively instead of (17) acceleration will be twice as much

$$\frac{dv}{dt} = g \sin \alpha. \quad (19)$$

Thus, descent from an identical hill brings in the considered cases to absolutely different speeds. Fairly is the converse: having originally identical speeds, *above all the wave on a rope will raise on a hill*, much lower - a rolling wheel, and most low "will climb up" the wheel if it slides.

Therefore it is logical to suggest that *soliton inertial properties are defined by internal processes*. Is not an exception and rotating electromagnetic soliton (the most elementary particle of substance). Under influence of gravitational or electromagnetic field internal processes change undergo in soliton. These inertial processes underlie also gravitational properties of substance and the effects described by the theory of relativity.

In particular, to explain mass of soliton Higgs's boson is hardly necessary to use. Everything is determined by structure and internal soliton processes!

Soliton on a rope: exact solution

Now let us try to find the exact solution for a single wave on a rope which properties became much clearer after rough calculations and mental experiments we have done.

To make necessary calculations and to get the corresponding differential equation we will use the balance of forces acting on a very small (elementary) part of a rope. But before it is necessary to describe entry conditions and starting positions which will become a starting point for solution search.

First, the analysis will be done in coordinate system moving together with a wave, that is, the wave will be motionless in relation to us, and the rope will evenly move, bending around a wave at a rate of v .

Secondly, we will begin with some initial angle α_0 , characterizing a steepness of leading (falling) edge of a wave at its basis (fig. 10).

After the solution corresponding to this steepness of a wave will be received, we can pass to the second stage - to search real angle α_0 matching the balance of kinetic energy and the potential energy in the entire wave.

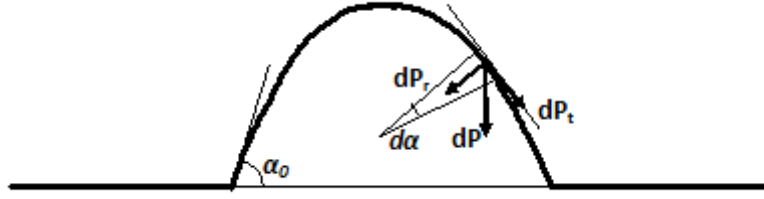


Fig. 10:
Tangential and radial components of weight dP
of an elementary rope piece.

First of all, we can see that below at the basis of a wave the tension of a rope T is equal to zero. Really, if it would not be so, the wave would have greater height, "raising" additional piece of a rope of the corresponding weight.

The very first from the bottom of a wave elementary rope piece dl has weight $mgdl$ with tangential P_t and radial P_r components (fig. 10). Let us see how these components are balanced by other acting forces.

So, radial component of weight

$$dP_r = mgdl \cos \alpha_0 = mg(Rd\alpha) \cos \alpha_0 \quad (20)$$

is counterbalanced by the corresponding centrifugal force

$$F_c = \frac{m(Rd\alpha)v^2}{R} = mv^2 d\alpha. \quad (21)$$

Here the length of elementary piece dl is expressed using radius of curvature R of a rope in this place and the corresponding elementary change of angle $d\alpha$.

Equating (20) and (21), we get the first elementary angle change

$$d\alpha_1 = \frac{mgdl \cos \alpha_0}{mv^2}. \quad (22)$$

Besides, there also appears the small tension of a rope equal to a tangential component of weight of the first elementary piece

$$T = dT_1 = dP_t = mgdl \sin \alpha_0. \quad (23)$$

This first step, perhaps, is the most difficult, because all further steps are made simply "automatically".

The difference is only that the following elementary pieces will get in addition influence of rope tension which (simultaneously with a radial component of weight) will counterbalance centrifugal force

$$mv^2 d\alpha = mgdl \cos \alpha + Td\alpha. \quad (24)$$

Therefore instead of (22) we will have

$$d\alpha = \frac{mgdl \cos \alpha}{mv^2 - T}. \quad (25)$$

It means the reduction of a steepness of a wave at the next small step dl along a rope, and respectively projections of weight (radial and tangential) of the following elementary piece (as well as rope tension) are changing. After these corrected values are obtained, all appears ready for the following step of calculations, and we move ahead making on a rope one more step.

Thus, step by step, we receive all parameters of a wave interesting us: wave form, its general height and force of a rope tension. To create corresponding data curves it is much simpler not to add elementary pieces of length in consecutive order with the corresponding angles, but to use projections to vertical and horizontal axes

$$dx = dl \cos \alpha, \quad dy = dl \sin \alpha. \quad (26)$$

Such calculations were made using the Microsoft Excel program. As concrete basic data the following figures were taken:

$$dl=0,1, \quad m=1, \quad v=10, \quad g=10, \quad \alpha_0=\pi/3.$$

After description of a wave form the second stage of calculations was carried out: calculation of potential and kinetic energy of each elementary rope piece and then corresponding total values of the entire wave were obtained. Further, with trial and error method it is possible to determine the value of α_0 assuring equality of total potential energy and total kinetic energy of a wave. Thus, all key parameters of the solitary wave on a rope, stretched on the ground, were defined.

In particular, it appeared that angle α_0 is about 73° , and that speed is connected with height and free fall acceleration by a ratio

$$v \approx 1,184 \sqrt{gH}. \quad (27)$$

As we see, the numerical factor in (27) really very slightly differs from approximate value 1,173 in formula (3).

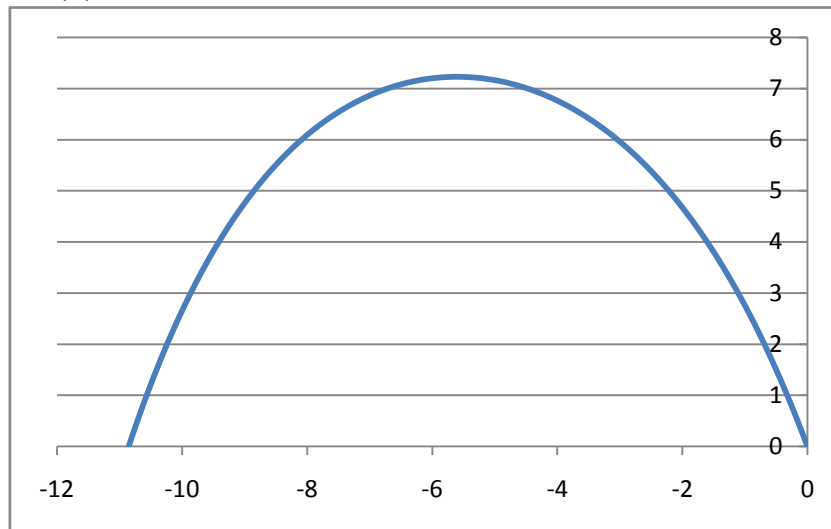


Fig. 11:
Solitary wave form $y(x)$ on the rope stretched on the ground.

Such approximate numerical methods leave sometimes feeling of disappointment as habitually we aspire to obtaining the analytical solution. But actually we have already entered for a long time an era of computer calculations which in practice proved the reliability and possibility of not less evident representation of results.

However, for fans of analytical calculations we also will draw up the differential equation describing a single wave on a rope.

And now let us look at the results of calculations.

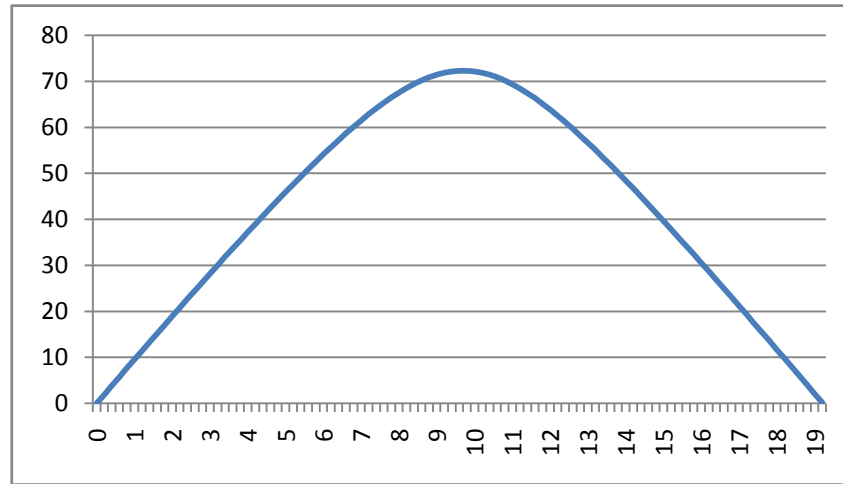


Fig. 12:
Rope tension T (l) along a wave length.

In fig. 12, unlike fig. 11, abscissa axis represents the length of a rope within a wave, instead of coordinate x . Therefore values on a horizontal axis are different.

To draw up differential equation for a solitary wave on a rope, we will use formulas already received earlier (20) - (26).

Taking into consideration formula (26) we will copy formulas (20) and (23) in a form:

$$\begin{aligned} dP_t &= mgdl \sin \alpha = mgdy, \\ dP_r &= mgdl \cos \alpha = mgdx. \end{aligned} \quad (27)$$

So the rope tension T is the sum of tangential components of weight of elementary rope pieces dP_t (23)

$$T = mgy. \quad (28)$$

Respectively the formula (25) takes on a form:

$$d\alpha = -\frac{mgdx}{mv^2 - mgy} = -\frac{dx}{\frac{v^2}{g} - y}. \quad (29)$$

As

$$\frac{dy}{dx} = tg \alpha, \quad \frac{d^2 y}{dx^2} = (1 + tg^2 \alpha) \frac{d\alpha}{dx}, \quad (30)$$

and taking into consideration (29), expression (30) will look like

$$\frac{d^2 y}{dx^2} = -\left(1 + \left(\frac{dy}{dx}\right)^2\right) \frac{1}{\frac{v^2}{g} - y}. \quad (31)$$

Finally desired differential equation looks so:

$$\left(\frac{v^2}{g} - y\right) \frac{d^2 y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + 1 = 0. \quad (32)$$

The solution of this equation, obviously, is identical to already obtained result. However, the calculations carried out earlier, are more informative, as contain data on some essential parameters.

For example, very important is information on a rope tension (fig. 12) as this parameter answers to a question about soliton stability. Really, any instability, arising in a wave, cannot go outside the wave limits as the rope tension monotonously decreases to zero from the wave center to the periphery. Thereof *any perturbing factor, coming nearer to the wave periphery, loses speed and cannot leave wave limits.*

One more important feature of the solution is realization with good accuracy of the above mentioned condition (7):

$$Lmg = 2mv^2 \sin \alpha_0, \quad 192 \approx 191,08. \quad (33)$$

This balance of forces is written down for the lower part of a wave where (as it was already mentioned above) there is no rope tension. As to accuracy of calculations, it was quite simplified modification: length of a step $dl=0,2$ at length of a rope $L=19,2$, that is, only 91 steps.

And at last, we should mention other party of the balance of forces (33) at the basis of a solitary wave. It is known that «action is equal to counteraction», and to counterbalance the "elevating" force there will be the force acting down. As a result of this back reaction the wave puts pressure with all its weight upon the ground, that is, soliton is not at all something "weightless".

Absolutely in a similar way presses on the ground already mentioned flexible ring which quickly rolls and keeps due to it almost round form. It would seem, the tension lifts a wheel, but it does not press it to the ground (the rope really cannot transfer compressing efforts). Nevertheless, the weight of a wheel affects the ground just with means of the back reaction arising in a place of a deformation of a wheel at contact with the ground (fig. 4). But in this case, unlike a solitary wave, it is necessary to take into account that the rope tension of «a flexible wheel» in the lower part is not equal to zero by a fast rolling.

Let us also notice that friction on the ground «holds down freedom» of a wave, but on a slippery surface it could not exist at all. It would be deformed and collapsed! Thus, the considered phenomena meeting all soliton features, in practice can be realized only in special conditions.

In other words, we investigated artificially supported phenomenon. But we also were convinced that this simple *soliton model* is very effective and evident as means for the demonstration of solitary wave properties.

Soliton on shallow water

How above received regularities for solitary waves on a rope can be useful by analysis of solitons on water? How much are these physical phenomena alike?

Similarity is rather obvious, as these waves can exist only thanks to gravitation, and respectively formulas for speed of movement of these waves appear very similar at each other (speed is approximately proportional to a root of wave height).

However, internal soliton dynamics on shallow water is much more complicated. For this reason Russell's ideas met misunderstanding even from outstanding scientists of that time Erie and Stoks, and the most exact description of the phenomenon was made only in 1895. It was done by the Dutch scientists Diderik Iokhannes Kortevæg and his pupil Gustav de Frieze [4]. However, their research and the equation received as a result with is called now *KDF - equation*, too appeared to be forgotten for almost 70 years. Moreover, the main property of a lonely wave (to act like a particle) for a very long time remained unnoticed as everybody, including Russell, considered it only as a wave. And only in 1965 Americans M. Kruskal and N Zabusky entered *the term "soliton" stressing similarity of this wave to a particle* [5].

Research of the mentioned main soliton property and soliton stability proof demanded gigantic efforts of physicists and mathematicians. And these efforts were rewarded: mathematics has got the new segment devoted to soliton, and physicists found out that the equations and properties of already known solitons can be spread to many other phenomena. In particular, it appeared that such different phenomena as impulses in nervous fibers and vortex in ideal water possess soliton properties.

The remarkable mathematical solitons description with the advent of the COMPUTER allowed to study in detail soliton properties. Mathematical modeling of processes including solitons collisions with each other is very interesting and admirable [6].

But obvious successes in the mathematical solitons description caused the aforesaid essentially pressed soliton representation, as physical phenomenon. Now soliton is known more through the mathematical equations describing this natural phenomenon. Therefore, just without belittling the mathematical aspect of a question, we would like, using minimum of mathematics, to look from the physical point of view at soliton on shallow water ("ancestor" of all solitons), so colorfully described by Russell.

Since school days many, probably, remember experiment with the water flowing in tube which diameter changes from smaller size to larger size and vice-versa (fig. 13). Pressure of liquid in pipe having bigger section appears essentially higher than in pipe with smaller diameter [7].

Such change of pressure upon transition of water flow to a big section is caused by sharply speed decrease (and upon transition to smaller section - by increase). Arising forces of inertia at change of water speed just cause the specified pressure difference.

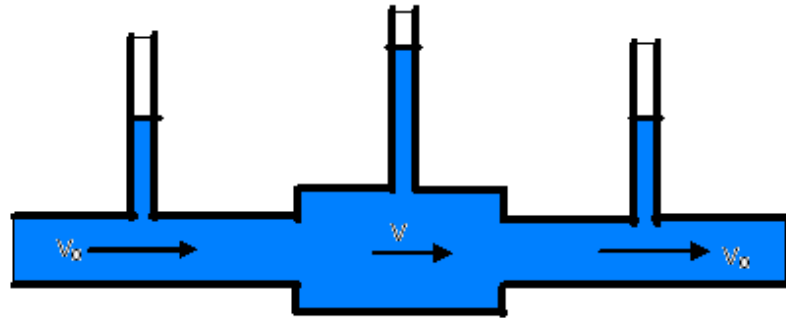


Fig. 13:
Pressure is higher where water speed is lower.

And now we will more attentively look what occur in a solitary wave on shallow water (fig. 14). Traditionally such experiments carry out in the trench of rectangular section filled with water. Moving together with the solitary wave along a trench with water, we will see that, in fact, it very much reminds water current in a pipe with changing cross-section. Really, in moving coordinate system we can believe that water moves by and where it «passes through a wave» cross-section appears increased, and speed of water respectively becomes less.

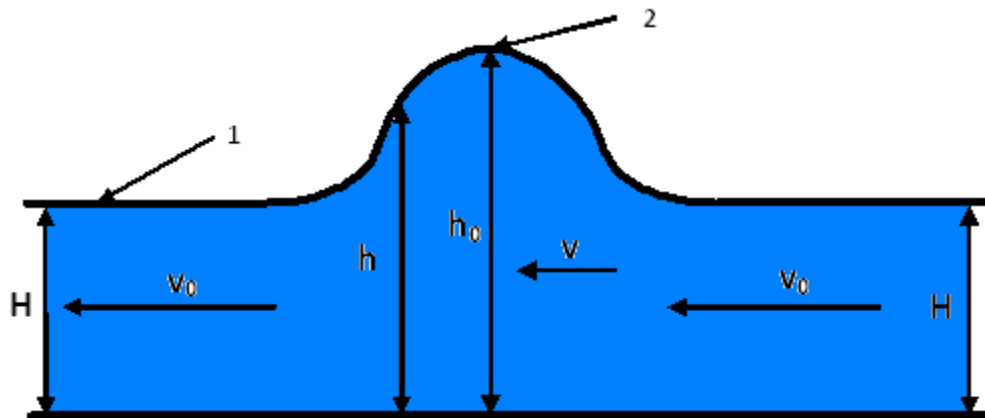


Fig. 14:
In moving coordinate system the wave is motionless,
and water "passes" by.

As well as in a case with a pipe pressure increases where speed of water is less. But, as a whole, processes in a wave appear more complicated.

Assumed that width of a trench is equal to one unit, the level of water is numerically equal to the cross-section (fig. 14). Thus through any vertical section water flow is the same (we move with a wave!)

$$Hv_0 = hv(h) = h_0v(h_0), \quad (34)$$

and current of water can be analyzed as a stationary current (invariable in time). Such current of ideal liquid (incompressible and no viscous) is described by Bernoulli equation familiar from school days [7]

$$p + \rho gh + \frac{1}{2} \rho v^2 = \text{const}. \quad (35)$$

Here p , h , v - pressure, height and speed of liquid in any point,
 ρ - liquid density.

Bernoulli equation is used for the solution of the most different problems of hydrodynamics. In particular, Torricelli's law (sizing up speed of the liquid flowing from a vessel through a small opening) from it directly follows

$$v^2 = 2gH. \quad (36)$$

Really, it is sufficient to write down the left side of equation (35) for two certain points in liquid and to equate these expressions to receive expression (36). The first point we select on a liquid surface in the top part of a vessel, and the second point - in flowing liquid near the opening at the outer side of the vessel (fig. 15).

$$p_1 + \rho gh_1 + \frac{1}{2} \rho v_1^2 = p_2 + \rho gh_2 + \frac{1}{2} \rho v_2^2 = \text{const}. \quad (37)$$

Pressure p in both points is approximately identical - atmospheric pressure, and the difference of heights of these points is equal to H . As regards the speed of level change of liquid in vessel v_1 , it is negligible under condition of opening smallness. As a result from (37) we receive expression (36). Obviously, *it expresses energy conservation law*.

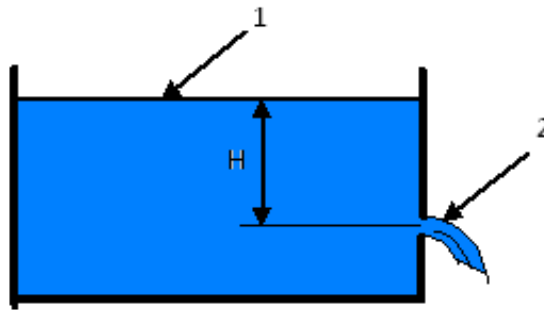


Fig. 15:
Two points choice to obtain formula (36).

Returning to soliton on shallow water, we will apply similar method and use Bernoulli equation to define characteristics of a wave. For drawing up the equation we will choose also two points on the water surface: one in unperturbed part of water where speed of water (in moving coordinate system) is v_0 , and the second point - on the top of a wave

where too there is no vertical component of speed (fig. 14). Similarly with the equation (37) we will write down the equation for two points

$$p_0 + \rho g H + \frac{1}{2} \rho v_0^2 = p_0 + \rho g h_0 + \frac{1}{2} \rho v^2. \quad (38)$$

Having substituted in (38) speed from (34)

$$v = \frac{v_0 H}{h_0}, \quad (39)$$

and considering that pressure p on the right and on the left sides of the equation (38) is almost identical and equal to atmospheric pressure, we will obtain the equation

$$g(h_0 - H) = \frac{v_0^2}{2} \left(1 - \frac{H^2}{h_0^2}\right). \quad (40)$$

In case of a low wave (h_0 little differs from H) from (40) follows approximate expression for speed of a wave

$$v_0^2 \approx g h_0 \left(1 + \frac{a}{2h_0}\right), \quad (a = h_0 - H). \quad (41)$$

The equation (40) can be also transformed to a quadratic equation in h_0

$$h_0^2 - \frac{v_0^2}{2g} h_0 - \frac{v_0^2}{2g} H = 0. \quad (42)$$

Solving the equation (42), we have

$$h_0 = \frac{H_0}{4} + \sqrt{\frac{H_0^2}{16} + \frac{1}{2} H_0 H}, \quad (H_0 = \frac{v_0^2}{g}). \quad (43)$$

In fig. 16 the dependence (43) is presented (for concrete values: $g=10$, $H=5$).

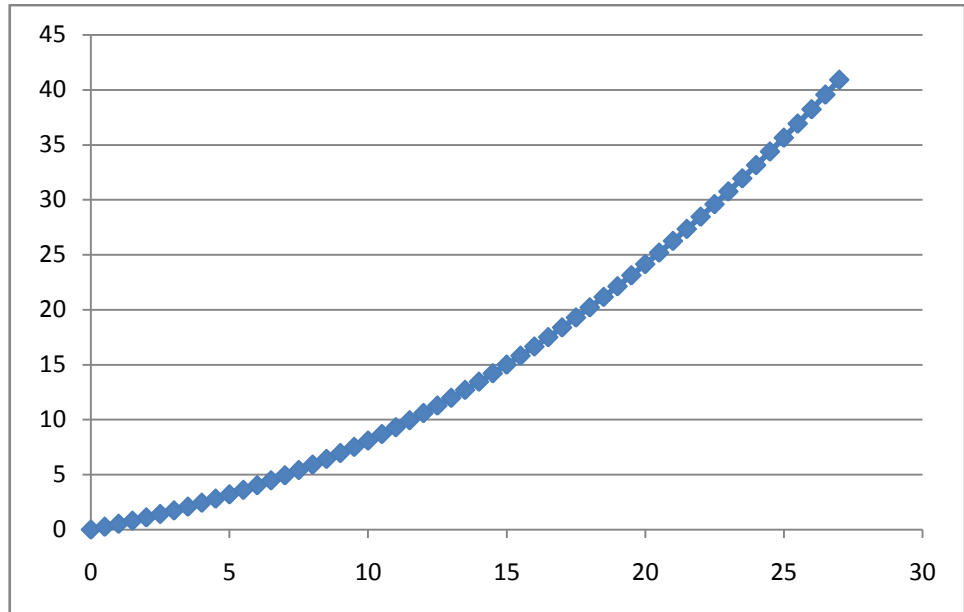


Fig. 16:
Dependence $h_0=h_0(v_0)$ when $g=10$, $H=5$.

Expression (43) can be used to specify conditions at which the single wave forms, having solved the corresponding inequation

$$h_0 > H, \quad \frac{H_0^2}{16} + \frac{1}{2}H_0H > (H - \frac{H_0}{4})^2, \quad H_0 = \frac{v_0^2}{g} > H. \quad (44)$$

How it is possible to interpret that result?

Actually we have got confirmation of *water real "tendency" to instability*.

Moreover, according to (44), to form soliton in a horizontal trench with motionless water, the wave is necessary to "accelerate" till speed v_0 exceeds *critical value* $(gH)^{1/2}$. Otherwise experiment most likely will not succeed.

On the other hand, formula (44) signifies that in a trench with flowing water *soliton is formed when speed of water current exceeds the same critical value* depending on depth of water

$$v_0 > \sqrt{gH}. \quad (45)$$

Therefore rapid water current seems as though it jumps up, making a start from «invisible bottom roughnesses», and forms almost motionless hillocks reminding solitary waves. But the experiment "purity" is essentially violated by water friction on walls and on trench bottom, and also by the turbulence influencing character of a current.

Let us pass to the more detailed description of a soliton form and soliton properties. First of all it is necessary to consider a vertical component of water speed on rising edge and falling edge of a wave (fig. 14)

$$\frac{dh}{dt} = \frac{dh}{dx} \frac{dx}{dt} = v \frac{dh}{dx} = \frac{v_0 H}{h} \frac{dh}{dx}, \quad (v = \frac{v_0 H}{h}). \quad (46)$$

Then the equation (38) for a slope of a wave will assume the following air:

$$g(h-H) = \frac{1}{2}(v_0^2 - (v^2 + \left(\frac{dh}{dt}\right)^2)), \quad (47)$$

$$2g(h-H) = v_0^2 - \frac{v_0^2 H^2}{h^2} \left(1 + \left(\frac{dh}{dx}\right)^2\right).$$

Let's re-arrange equation (47)

$$\frac{dh}{dx} = \sqrt{(v_0^2 - 2g(h-H)) \frac{h^2}{v_0^2 H^2} - 1}. \quad (48)$$

Formula (48) is convenient for calculation in the Microsoft Excel program. It was carried out to show up a wave form at various initial parameters.

What was sequence of calculations?

At first, using formula (43), we get coordinate of the highest point of a wave h_0 (for initial parameters: $v_0=10$, $g=10$, $H=5$).

Then, the first step follows: we decrease the received *value* h_0 by chosen stride parameter ($\Delta h=0,1$) and calculate derivative value (48). After that, knowing derivative value (C), it is possible to calculate corresponding change of horizontal coordinate

$$\frac{dh}{dx} = C, \quad \Delta x = \frac{\Delta h}{C}. \quad (49)$$

Thus, for construction of graph we have the second point $((h_0 - 0,1), \Delta x)$. Let's remind that the first point was $(h_0, 0)$.

The subsequent steps are made by simple filling of columns. With each step h_0 decreases by 0,1 and the derivative value, as well as corresponding change of horizontal coordinate Δx are calculated.

The result of calculation is presented in fig. 17. There is only a "half" soliton as the second slope is symmetric to the represented one, and there is no need to do the corresponding constructions.

As we see, the received curve smoothly approaches to the initial water level $H=5$ that indirectly proves the calculations. Thus accuracy of calculations appears quite acceptable even taking into account that (as well as in the previous calculations) rather few points were taken.

It is necessary to notice that the modern simplest personal computer gives excellent possibility not only to carry out rather difficult and bulky calculations, but also as we saw to solve the differential equations actually "without solving" them and to calculate any integral. Moreover, results of calculations can be right there "instantly" processed and presented in a very convenient form.

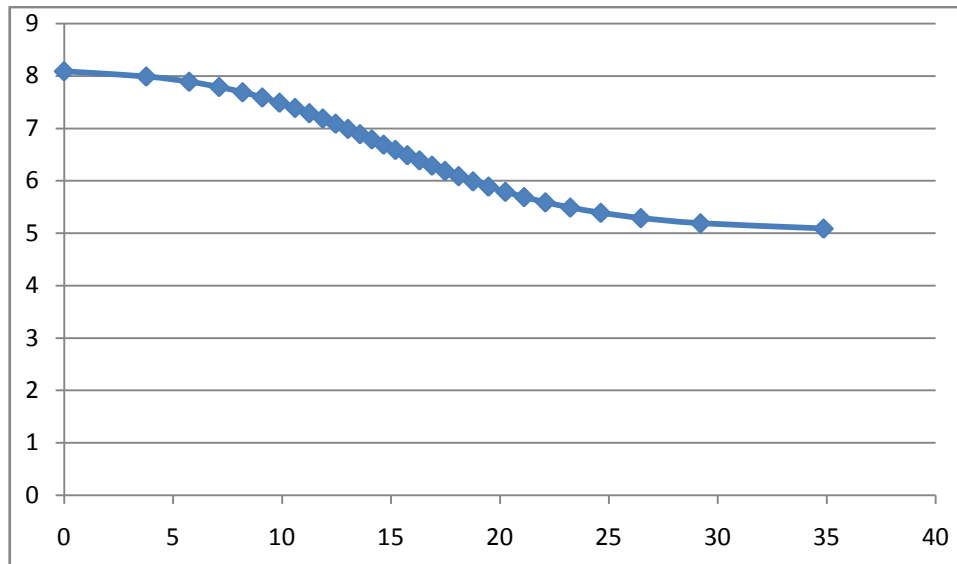


Fig. 17:
Dependence $h=h(x)$ provides insight into a soliton form
(right "half" of soliton is represented at $v_0=10$, $g=10$, $H=5$).

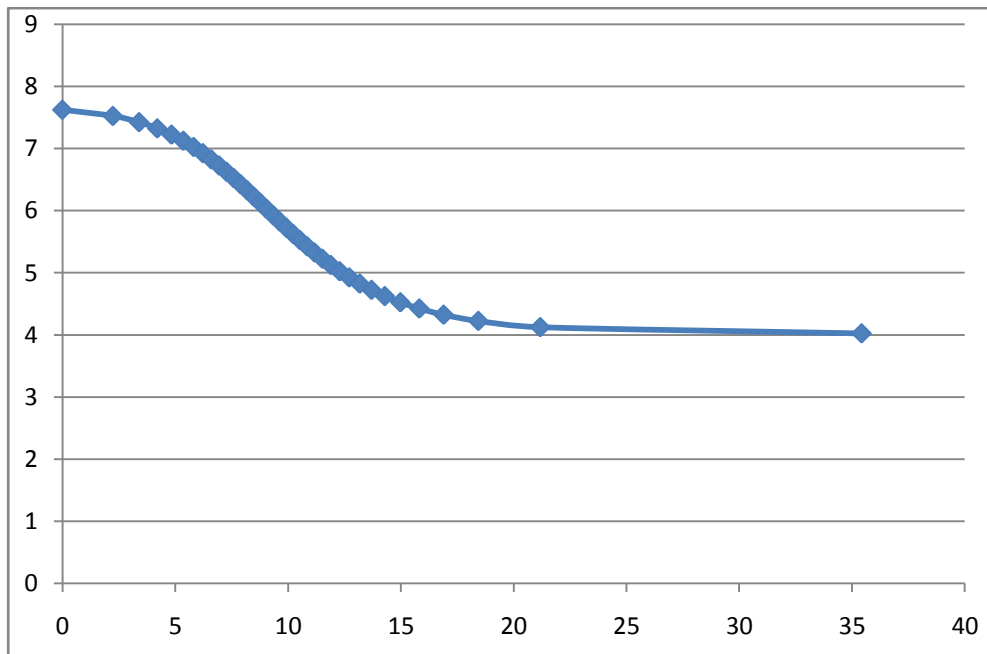


Fig. 18:
Dependence $h=h(x)$ at $v_0=10$, $g=10$, $H=4$.

Therefore wishing to study soliton solutions in more ample treatment, you can carry out the calculations using computer and make corresponding simple graph. Changing initial parameters, it is interesting to observe instantly changing characteristics and instant graph that looks like soliton "animation" on the monitor screen.

Such calculations give the chance to estimate visually influence of each factor on this unusual physical phenomenon. For example, in fig. 18 in comparison with fig. 17 depth of water is reduced from $H=5$ to $H=4$ under other equal conditions. Comparison of graphs shows that soliton slope steepness in fig. 18 is sharply increased, as well as wave height, but soliton width at once decreased.

Such sharp increase in height and steepness of a wave during movement to the coast when there is a reduction of water depth just indicates "crafty" property of a tsunami to fall over the coast with all might. Besides, as we saw (soliton on a rope), solitons climb up highly on a height, and in case of a tsunami they transfer with themselves great volume of water allied with the huge internal energy - equally both kinetic, and potential energy.

Let's note also sharp growth of height of a wave under reduction of acceleration of a free fall (it is useful to know for space travelers!).

It would be useful to look more attentively at the interaction between kinetic energy and potential energy in soliton. This process is directly linked with water vertical speed on wave slopes.

How it comes out? Deceleration of a water current at increase of cross-section of a stream, as it was already noted, leads to appearance of inertia forces and water pressure increases. Pressure upon a bottom respectively increases, "making a start" for water vertical up accruing speed.

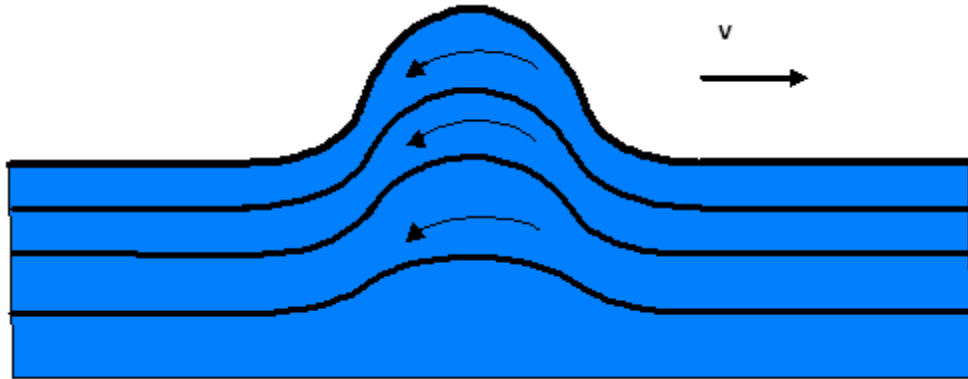


Fig. 19:
The wave can be mentally divided into layers,
following the water movement.

Thus, there is a transformation of kinetic energy of water to potential energy. On back slope of the soliton there is a return process: water goes down, and its potential energy comes back accelerating the stream.

Described dynamics of liquid can be in some way compared with soliton movement on a rope. Having mentally divided the stream into horizontal layers, following the water movement (fig. 19), we will see that forces operating in water can be interpreted as centrifugal forces due to water movement along curved trajectories. However, in this case centrifugal forces are not transformed to a tension (as on a rope), but obviously cause pressure change in liquid. The center of a wave turns into a powerful pump raising the water and holding it above (fig. 20).

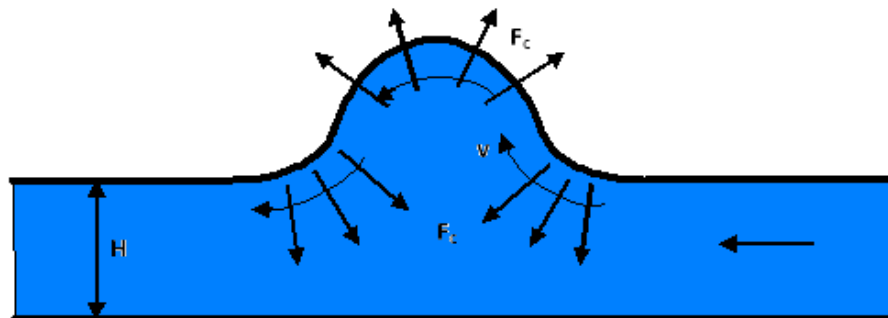


Fig. 20:
Centrifugal forces have opposite directions
at the basis and on the top of the wave.

Therefore, when soliton approaches the coast and water depth decreases, this enormous "pump" experiences the lack of water. Soliton draws in the remains of water

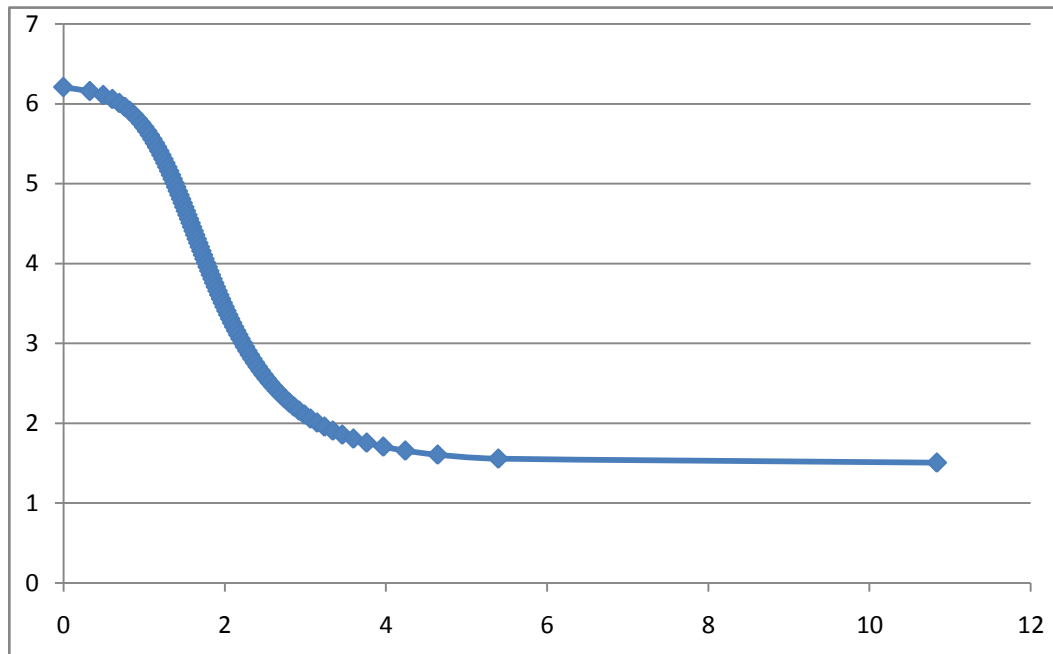
receding from the coast. Because of water lack the part of energy which was spent for water "pumping out" from depth, goes to increase wave height along with simultaneous reduction of wave length. This process ends with water collapse on the coast.

However the reader will reasonably notice that pressure decrease by centrifugal forces in the wave center is compensated by pressure increase because of accruing "thickening" of layers in the center (fig. 19). And it is true. Therefore we should "measure" real pressure under soliton (at the bottom) and for this purpose we have all possibilities.

After having defined the form of soliton and respectively the speed of water, we can set up an equation similar to the equation (47), but «the first point» for the equation must be again on a water surface away from soliton, and «the second point» - at the bottom direct under the soliton

$$p_1 + \rho gh + \frac{1}{2} \rho v_0^2 = p_2 + \frac{1}{2} \rho \frac{v_0^2 H^2}{h^2}. \quad (50)$$

Here it is considered that near the bottom (zero height) water has only horizontal component of speed (the right side of equation), as well as at the surface of undisturbed water (left side of equation).



a)

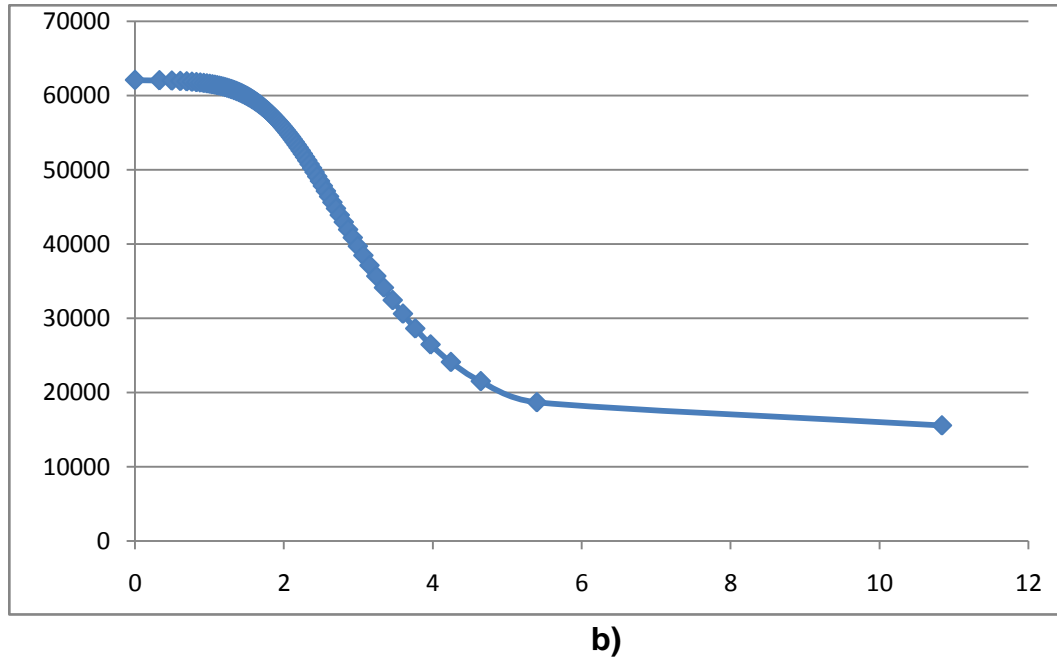


Fig. 21:
a) soliton shape, b) pressure at the bottom
(without atmospheric pressure), $H=1,5$; $\rho = 1000$.

As pressure at the water surface is equal to the atmospheric pressure, from (50) we obtain the following expression for pressure value at the bottom

$$p_2 = p_0 + \rho gh + \frac{1}{2} \rho v_0^2 - \frac{1}{2} \rho \frac{v_0^2 H^2}{h^2}. \quad (51)$$

All input values are specified in previous calculations therefore we need only to add in table one more column and to construct the pressure graph.

The effects caused by the vertical component of water speed, as it was mentioned above, become appreciable when soliton reaches small depth. In fig. 21 results of calculation of soliton form are given and pressure at the bottom direct under soliton when depth of water is 1,5. Besides, only water pressure is taken into consideration (without atmospheric pressure).

In graphs it is easy to notice that pressure under soliton grows obviously quicker, than the water height increases on the soliton slope. To estimate this effect, we will subtract pressure of water (ρgh) from a combined value of pressure (fig. 22).

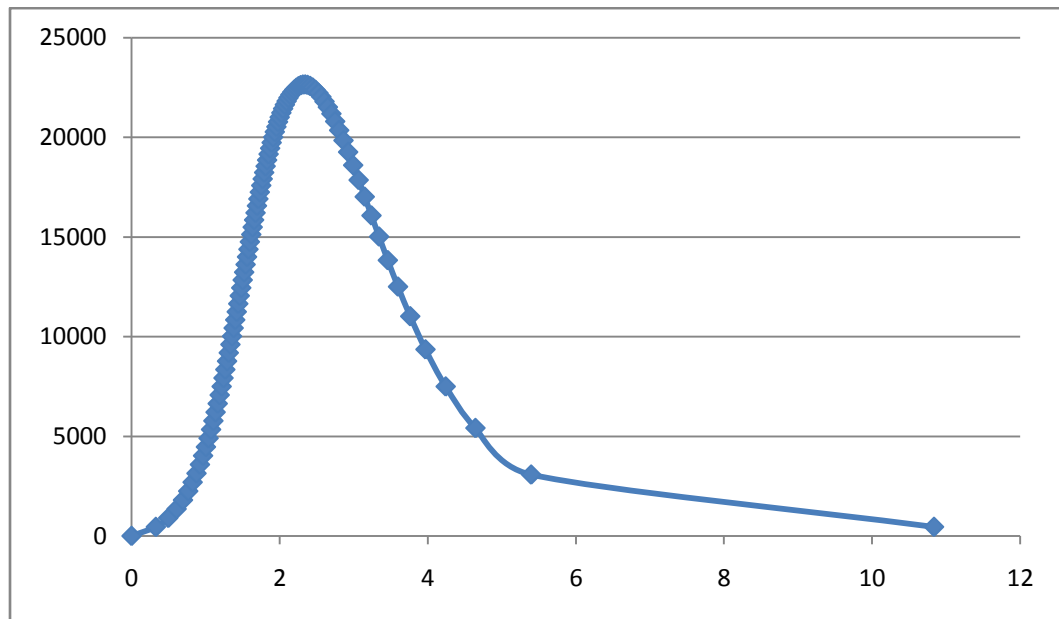


Fig. 22:
At wave front passing there is a sharp pressure jump,
considerably exceeding pressure of water column.

As we assumed, *on a rising edge of soliton sharp pressure jump appears on a bottom under solitons*, significantly exceeding pressure of a water column. Such feature of soliton on shallow water, undoubtedly, is the additional *factor increasing destructive force of a tsunami*.

Conclusions

Soliton (as well as any other physical phenomenon) "is absolutely indifferent" to whether it is analyzed in motionless coordinate system or in one moving together with it. However the analysis depends on the chosen coordinate system. It becomes not only simpler or more complicated, but can even change physical meaning of process and corresponding interpretation of received results.

Therefore we could see much general in solitary wave on the rope stretched on the ground and in dynamic instability of a flexible hose with flowing water. Unexpectedly similar there are processes in soliton on shallow water and in water stream through a pipe having variable cross-section. On the other hand, results of the analysis of soliton on motionless water are applicable for an explanation of instability arising in moving water.

We have seen that simple and evident models can be used for adequate analysis of complicated physical processes occurring in solitons. Physical aspect of the phenomena interested us most of all, but not to oppose to a mathematical approach to soliton study. On the contrary, we undertook this attempt to make the phenomenon description more volumetric.

It is known that soliton really can be identified by its time change and by interactions with each other. These particularly important properties are enclosed in the "soliton equations" with time as parameter. It gives possibility for computer simulation of such processes that is necessary for classification and soliton study.

At the same time, solitons are the steadiest physical objects very slowly changing with time. Therefore it was convenient to use this additional possibility in details and more attentively to study soliton "portrait". Actually, having excluded development with time, we considered processes as *stationary*.

It is current opinion that solitons arise in nonlinear medium. And in many cases it is really so. However there are not so obvious cases.

What nonlinearity causes soliton on shallow water? Actually soliton itself creates this nonlinearity! Water is homogeneous isotropic incompressible and not too viscous medium. Only soliton, "organizing" in a special way interaction of potential and kinetic kinds of energy, makes this medium "nonlinear". As a result this complicated dynamic system passes to a *special steady energy state*.

What are the conditions for soliton formation? This being self-supported process appears "viable" first of all because of being "well-founded" as power object. And *"nonlinearity" is generated by the physical phenomenon itself and to it corresponds*, but not the reverse. After all water movement can have a set of other shapes (including, vortex cavity) at which there are absolutely others "nonlinearities".

The description of soliton on shallow water using analogy with a stationary current of liquid essentially simplifies the task and gives the chance to apply Bernoulli equation which expresses *equality of the specific energy falling per unit volume of liquid in all points of a stationary current. The stationary current is supported by that any deviation breaking specified equality, leads to a gradient of energy which, in turn, eliminates the occurred deviation*.

The equation (40), its solution (43), and also solutions for all points of the slope of a solitary wave testify that *soliton is only one of possible steady state options*. Moreover, we managed to find conditions at which it is realized in motionless and flowing water. It means that other steady states modes (with a water level h_0 , H , or in the form of a step) are also possible.

Quite often a question arises why on shallow water it is not observed soliton in the form of a hollow – figuratively saying, anti-soliton? There is the same water "nonlinearity"! The answer is the solution (43) of equation (40): *the soliton modification in the form of a hollow provided to be unrealizable for energy reasons*. If to take a minus sign before a square root in (43), negative value comes out. Therefore such soliton version in practice is not observable.

As well as in a case of soliton on water, it is probable to ask a question what "nonlinearity" is comprised in a rope as "medium" for wave process. It is obvious that the rope per se does not possess such nonlinearity. Everything depends on companion conditions (for example, whether the rope is spread out on the ground, or it is drawn between two rigid supports) and what processes are initiated in this "medium". Only then

nonlinearities arise which correspond to the «advancing process». That is, one cannot be separated from another. They do not exist one without another! Solitary wave on a rope, for example, is stable due to *monotonous reduction of a rope tension from the wave center to the periphery; it is the real physical explanation of process stability*.

The represented medium examples, in which solitons arise, illustrate only a general principle operating in the nature: Her Majesty Energy "invented" incalculable quantity of self-organizing systems in different mediums. But main among them, undoubtedly, is the all-inclusive medium – “physical vacuum”. And we have no right not to mention about it.

In above considered mediums two kinds of energy coexist and interact thanks to gravitation, but vacuum does not need such "help". Vacuum can accumulate simultaneously two kinds of energy: electric (analog of potential energy) and magnetic (analog of kinetic energy). Mutual transformation and interaction of these two kinds of energy create “favorable conditions” for solitons (elementary particles) in vacuum.

But what is about nonlinearity necessary for solitons existence in vacuum? After all it is well known that the vacuum is isotropic medium, is an ideal insulator and has characteristics –permittivity and permeability of free space, which are real numbers. Nevertheless, soliton again «found a way to obtain» nonlinearity of vacuum characteristics!

Soliton "profited" by rotation of an electromagnetic field, which creates specific tension in vacuum generating nonzero electric field divergence (charge). In turn, spatial distribution of this charge makes soliton to rotate, creating a radial gradient of propagation speed of electromagnetic field within soliton structure [8-10]. So the integral soliton property - self-organizing shows itself!

It is interesting that in soliton structures with rotating electromagnetic field (electron and other elementary particles) there are areas having positive charge, as well as areas with negative charge. Charged particles have only some overweight either negative or positive charge. For example, in positron - electron antiparticle (all fields in their structures have opposite directions), the positive charge prevails over a negative charge, and in electron respectively negative charge dominates.

To "see" all these processes in vacuum is possible resorting to help Maxwell's model [11]. The vacuum model appears much more substantial than electromagnetic field equations derived by Maxwell on its basis!

Characteristic soliton feature is equal distribution of its internal energy between two energy kinds (potential and kinetic, electric and magnetic). An electron, as well as any soliton, comprises equally electric and magnetic energy. Precisely so - equally energy is distributed between these two kinds when there is an increase in electron speed accompanied with the corresponding accumulation of internal energy. Convincing confirmation to it is photon radiation (having equal quantities of electric and magnetic energy) at sharp electron braking when it "gets rid" of stored energy.

Why soliton as phenomenon is so important for physics? It is the best keeper and «the soldier» protecting energy and, at the same time, it is the best energy embodiment. And it is not exaggeration!

We saw, how carefully soliton bears energy and with firmness "fights" to survive coming nearer to the coast (soliton on shallow water), or overcoming undulations of ground (soliton on a rope).

Therefore it is no coincidence that solitons are used extensively in means of communication as *solitons pass many thousands kilometers, struggling with its "enemies" - dissipation and dispersion!*

However nothing can be compared with real wonder of the Nature - electromagnetic solitons in vacuum [12]. These are true *keepers of the Universe* because they bear and keeps energy for *many billions years*, protecting it from spreading and smearing in space and defining thus the world we know.

In particular, theory of relativity does not reflect properties of space; on the contrary, it reflects solitons properties making substance. Such point of view is not only assumption – it also finds the evidence in theoretical researches [13, 14].

The tremendous density of energy which solitons (elementary substance particles) contain speaks about the "tough" past of the Universe. These solitons, living already many billions years (as well as others solitons - relic photons), are witnesses and direct "participants" of events of that incredibly far era.

Electromagnetic soliton in vacuum, described by Maxwell model, gives the opportunity to explain properties and all characteristics of the simplest particle of substance. Even the first outlines of the soliton theory of the electron I made in 1986, surprised me by opening possibilities in the nature description, by singularity and beauty of this physical phenomenon.

However, then I yet did not know even the word "soliton". When I showed my draft of an electron to the remarkable person and the talented scientist, Doctor of Engineering Mikhail Borisovich Golant, he perused it surprisingly carefully and told that all this *is very like soliton*. Only after that I started to be interested in this strange, as it seemed to me, phenomenon. Now I see, how great was the intuition and scientific erudition of M. B. Golant. He is gone, but many people and I among them, with gratitude remember the remarkable scientist and true character, the war veteran.

In this connection I should remember excellent creative atmosphere, which reigned in those years in research institute "Istok" (Fryazino) – headquarters plant of the Ministry of Electronics Industry of the USSR. To list here all leading scientists of the powerful scientific school of this remarkable institution it is impossible. Here are only few names. V. F. Kovalenko - the inventor of reflex klystron and the most talented scientist was my scientific advisor. The most direct and active participation in my scientific destiny took remarkable scientists: N. V. Cherepnin - prominent founder of manufacturing techniques of electronic devices, the author of remarkable books in this domain of science; L. A. Paryshkuro - talented developer and organizer of science, the Hero of Socialist Labor; V. P. Sazonov, R. A. Silin, V. S. Lukoshkov, A. S. Pobedonostsev - classical theorists of the microwave electronics and many others. The Academic council of the research institute was headed by academician N. D. Devyatkov. It is natural that the fact of defense of dissertation (1984) in such Academic council per se was for me the greatest responsibility and the severe test.

One more remarkable enthusiast of a science with whom I had occasion to make acquaintance was the editor-in-chief of the popular science magazine "Eretik" V. G. Denisenko who published in 1991 my article «Mystery of electron», containing soliton version of an electron structure [15].

In conclusion I would like to express hope that new generation of physicists and mathematicians will make every effort to study the most widespread solitons in nature and major in the soliton science - elementary particles of substance.

Maxwell's vacuum is not only the electrodynamics equations, but also solitons existing in this surprising universal medium.

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